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Natural convection heat transfer from helical coiled tubes

Devanahalli G. Prabhanjan, Timothy J. Rennie, G.S. Vijaya Raghavan *

Department of Bioresource Engineering, Macdonald Campus of McGill University, Ste. Anne-de-Bellevue, QC H9X 3V9, Canada

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Abstract

An experimental investigation of the natural convection heat transfer from helical coiled tubes in water was performed. The outside Nusselt number was correlated to the Rayleigh number using different characteristic lengths. The relationship was based on a power law equation. The constants in the equation are presented for each of the different characteristic lengths used. The best correlation was using the total height of the coil as the characteristic length. The developed models were then used to develop a prediction model to predict the outlet temperature of a fluid flowing through a helically coiled heat exchanger, given the inlet temperature, bath temperature, coil dimensions, and fluid flow rate. The predicted outlet temperature was compared to measured values from an experimental setup. The results of the predicted temperatures were close to the experimental values and suggest that the method presented here has promise as a method of predicting outlet temperatures from similarly dimensioned heat exchangers.

Keywords: Helical pipe; Nusselt number; Natural convection; Heat exchanger

1. Introduction

Helically coiled tubes are effective as heat transfer equipment due to their compactness and increased heat transfer coefficients in comparison with straight tube heat exchangers. Helical coils are used for heat exchange in the fields of air conditioning, nuclear power, refrigeration, and chemical engineering [1].

Developing fluid-to-fluid helical heat exchangers (fluid is present on both sides of the tube wall) requires a firm understanding of the heat transfer mechanism on both sides of the tube wall. Though much investigation has been performed on heat transfer coefficients inside coiled tubes, little work has been reported on the outside heat transfer coefficients. Ali [2] obtained average outside heat transfer coefficients for turbulent heat transfer from vertical helical coils submersed in water. In these experiments water was pumped through the coil and the inside heat transfer coefficients were calculated based on the Nusselt number correlation of Rogers and Mayhew [3]. Outside heat transfer coefficients were calculated based on the thermal resistance method for cylindrical tubes. Five different pitch-to-helical diameter ratios were used, along with two tube diameters and with different numbers of turns. The outside Nusselt number was evaluated using the length of the tube, L, as the characteristic length. The Nusselt number was related to the Rayleigh number using the following relationships for outside diameters of 0.012 and 0.008 m, respectively.

$$Nu_{\mathrm{L}} = 0.685(Ra_{\mathrm{L}})^{0.295} \qquad 3\times10^{12} \leqslant Ra_{\mathrm{L}} \leqslant 8\times10^{14} \tag{1}$$

$$Nu_{\mathrm{L}} = 0.00044(Ra_{\mathrm{L}})^{0.516} \qquad 6\times10^{11} \leqslant Ra_{\mathrm{L}} \leqslant 1\times10^{14} \tag{2}$$

Ali [2] stated that from the observations, h_{o} decreases slightly with boundary layer length for an outside diameter of 0.012 m while it increases rapidly with the boundary layer length for a diameter of 0.008 m. Ali [2] also suggested that increasing the tube diameter for the same Rayleigh number and tube length will enhance the outer heat transfer coefficients. However, Xin and Ebadian [1] state that the large behavioral differences between different tube diameters in Ali's [2] experiments are inexplicable. Despite using different pitches, none of Ali's [2] correlations took the pitch into consideration. Ali [2] also used the data to develop the Nusselt relationship with the Rayleigh number using the height, H, as the characteristic length of the coil,

* Corresponding author.
E-mail addresses: timothy.rennie@mail.mcgill.ca (T.J. Rennie), raghavan@macdonald.mcgill.ca (G.S. Vijaya Raghavan).

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Nomenclature

a	empirical constant	Pr	Prandtl number
A_o	outer surface area . . . . . m^2	q	heat transfer rate . . . . . W
A_i	inner surface area . . . . . m^2	r_i	inner tube radius . . . . . m
b	empirical constant	r_o	outer tube radius . . . . . m
c_p	specific heat . . . . . $J \cdot kg^{-1} \cdot K^{-1}$	R	coil radius . . . . . m
D	tube diameter . . . . . m	Ra	Rayleigh number
dT_w	temperature difference across tube wall . . . . . K	Ra_L	Rayleigh number based on tube length
H	height of coil . . . . . m	$Ra_{o.d.}$	Rayleigh number based on tube outer diameter
h_i	inside heat transfer coefficient . . . $W \cdot m^{-2} \cdot K^{-1}$	Re	Reynolds number
h_o	outside heat transfer coefficient . . $W \cdot m^{-2} \cdot K^{-1}$	$Relax$	relaxation factor
k	thermal conductivity . . . . . $W \cdot m^{-1} \cdot K^{-1}$	t	tube wall thickness . . . . . m
L	tube length . . . . . m	T_{bath}	bath temperature . . . . . K
L_N	normalized coil length . . . . . m	T_{bulk}	average bulk temperature . . . . . K
Nu_i	inside Nusselt number	T_{Fi}	inside film temperature . . . . . K
Nu_H	outside Nusselt number based on coil height	T_{Fo}	outside film temperature . . . . . K
Nu_L	outside Nusselt number based on tube length	T_{in}	inlet temperature . . . . . K
Nu_o	outside Nusselt number	T_{out}	outlet temperature . . . . . K
$Nu_{o.d.}$	outside Nusselt number based on tube outer diameter	T_w	average wall temperature . . . . . K
		T_w^*	newly calculated average wall temperature
$\dot{m}$	mass flow rate . . . . . $kg \cdot s^{-1}$	U_o	overall heat transfer coefficient . . . $W \cdot m^{-2} \cdot K^{-1}$

which considers both the pitch and the tube diameter. For an outer tube diameter of 0.012 m and for curvature ratios (ratio of tube radius, r_o, to coil radius, R) of 0.048, 0.072, and 0.101, the best power law fit obtained was

$$Nu_H = 0.257(Ra_H)^{0.323} \qquad 6 \times 10^8 \leqslant Ra_H \leqslant 3 \times 10^{11} \tag{3}$$

As the exponent is just under 1/3, it indicates that h_o is decreasing very slightly or that it may be constant along the coil height [2]. For a tube diameter of 0.008 m, and curvature ratios of 0.050 and 0.101, two distinct regions were obtained: one for laminar flow and the other for turbulent flow. Correlations were obtained for the transition region for both curvature ratios. Interestingly, Ali [2] considered that there was a laminar, transitional, and turbulent regions in his experiments for an inner diameter of 0.008 m and correlated only for the transitional region. For the correlation of Nu_L the same data was used but it was not divided into the three zones, all zones were considered in one correlation.

Xin and Ebadian [1] used three different helicoidal pipes to determine the outside heat transfer coefficients for natural convection. The coils were oriented both vertically and horizontally. The tube wall was heated by passing a high dc current through the tube wall, resulting in a constant heat flux boundary condition. The relationship of the Nusselt number, as a function of the Rayleigh number, was based on the outer diameter of the tube. The outer heat transfer coefficient was based on temperature measurements on the outside of the tube. The average Nusselt number was correlated with the Rayleigh number for the vertical coils as [1]:

$$Nu_{o.d.} = 0.290(Ra_{o.d.})^{0.293} \qquad 4 \times 10^3 \leqslant Ra_{o.d.} \leqslant 1 \times 10^5 \tag{4}$$

It should be noted that this correlation are not directly compared to those of Ali [2], as the characteristic length of Ali [2] was the length of the coil and the height of the coil, and not based on the outer diameter of the tube.

For the case of a horizontal coil, local Nusselt numbers were higher on the top and the bottom of the coil than on the sides. The average Nusselt number correlation obtained was [1]:

$$Nu_{o.d.} = 0.318(Ra_{o.d.})^{0.293} \qquad 5 \times 10^3 \leqslant Ra_{o.d.} \leqslant 1 \times 10^5 \tag{5}$$

The correlations of Xin and Ebadian [1] show that the average heat transfer coefficient of the vertical coil was about 10% higher than for the horizontal coil in the laminar flow regime.

Ali [4] criticized the work of Xin and Ebadian [1] stating that their correlation for the horizontally orientated coil was not useful for practical applications as the correlation did not take into account the end effects which would be important to consider in real applications. Ali [4] performed experiments to measure the average Nusselt number for the whole coil, including end effects, for a coil with a constant heat flux. The correlation between the Nusselt number and the Rayleigh number was based on the outer tube diameter as the characteristic length. Ali [4] used four different coils, each with the same tube diameter but with two different helical diameters and with different number of turns.

Ali [4] correlated the Nusselt number as a function of the Rayleigh number for each of the different heat fluxes used. It was found that the Nusselt number decreased with increasing Rayleigh numbers.

In all, the number of studies on the outside natural convection heat transfer is not sufficient to properly design heat exchange equipment and there exists a need for more studies involving different helical configurations and flow rates.

2. Objective

The objective of this study is to:

(1) determine the Nusselt number correlation for natural convection from a vertical helical coil;
(2) attempt to correlate the Nusselt number with dimensionless numbers that represent the flow and heat transfer around the outside of the coil;
(3) develop a prediction model to evaluate the outlet temperature from a helical coil with natural convection heat transfer from the outer surface.

All objectives are to be preformed on a helical heat exchanger where the processing fluid is pumped through the tube and the carrier fluid is unmixed.

3. Experimental setup

The physical dimensions of the four coils that were used are given in Table 1. Each coil was made of copper and consisted of $9\frac{1}{2}$ turns. The pitch is defined as the distance between the centerline of the tube coil for two subsequent turns. Coils 1, 2, and 3 were used to develop Nusselt–Rayleigh correlations. Coil 4 was used to validate these correlations. The fluid was pumped through the coils using a positive displacement pump connected to a variable speed motor. The speed of the motor was adjusted to obtain flow rates of 0.10, 0.15, and 0.20 $kg{\cdot}s^{-1}$. These correspond to a turbulent flow regime with Reynolds numbers in the range of 12 000 to 27 000. The purpose of using different flow rates was to change the temperature distribution along the tube wall and to change the inside heat transfer coefficient. Water

Table 1
Coil dimensions used in the experiment

Coil	Tube diameter (mm)	Helix diameter (mm)	Pitch (mm)
1	15.8	305	47.4
2	15.8	305	15.8
3	13.5	203	13.5
4	13.5	203	40.5

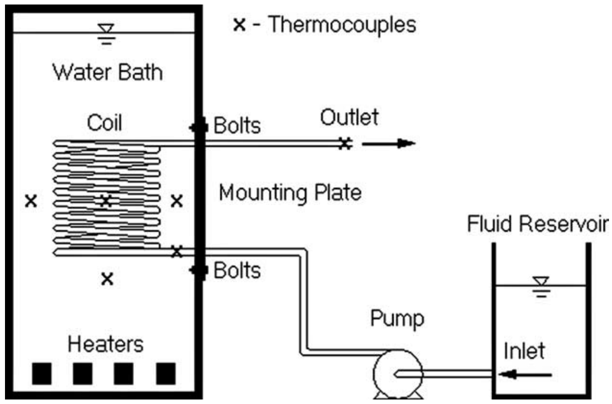

Fig. 1. Experimental setup.

(19 °C) was used as the inlet fluid and came from a reservoir that was being constantly recharged. In total there were 24 tests (4 coils, 3 flow rates, and 2 water bath temperatures).

For each experiment, the required coil was mounted on a rectangular mild steel plate with swage lock fittings. The inner diameter of the fittings was equal to the inner diameter of the coil to prevented disturbance of the flow pattern of the fluid when entering and exciting the coils. Rubber gaskets were glued on both sides of the mounting plate.

A large water bath was used to house the coils. It was rectangular and made of 20 gauge galvanized iron sheet. The dimensions were 600 × 600 × 1200 mm (Fig. 1). It had a rectangular cutout were the abovementioned mounting plate was set into. Four 5000 W electrical heaters were fixed at the bottom. Two were on all the time and two others were used as needed to maintain a constant water bath temperature (75 or 90 °C). The water bath was insulated with a 50 mm thick polyurethane foam (R-10), covered with galvanized iron sheet.

Temperature measurements were made using type K (nickel–chromium vs. nickel–aluminum) thermocouples with 30-gauge extension wire. The temperatures were recorded with a DATAshuttle Express™ (StrawberryTree, Sunnyvale, CA) data acquisition system. This system had 16 analog inputs with a 13-bit resolution. Two thermocouples were used to measure the water inlet temperature and 4 for the water bath temperature. The outlet temperature was measured using a type-k thermocouple attached to a handheld temperature display. All thermocouples (Omega Engineering, Stamford, CT) were rated to meet limits of error of 2.2 °C or 0.75% whichever is greater. Temperature measurements were recorded at a rate of 1 Hz. However, the system was allowed to come to steady state before the data acquisition.

4. Calculation method for outside Nusselt number

The total amount of heat transferred, q, was calculated based on the mass flow rate, $\dot{m}$, the specific heat of the processing fluid, c_p, and the difference in inlet and outlet temperatures ($T_{in} - T_{out}$) given by

$$q = \dot{m} c_p (T_{in} - T_{out}) \tag{6}$$

All quantities were measured except the heat transfer rate. The heat transfer rate was then used to calculate the overall heat transfer coefficient, U_o,

$$U_o = \frac{q}{A_o \Delta T} \tag{7}$$

A_o is the outside surface area of the coil and ΔT is the average temperature difference between the bulk fluid in the coil (average of the inlet and outlet temperatures) and the fluid in the bath. The overall heat transfer coefficient can be described in terms of thermal resistances for a cylindrical tube as:

$$U_o = \frac{1}{[1/h_o + (r_o \ln(r_o/r_i))/k + r_o/(r_i h_i)]} \tag{8}$$

The radii for the inner wall and the outer wall of the tube are r_i and r_o, respectively, and the thermal conductivity of the coil is k. It was necessary to determine the inside heat transfer coefficient, h_i, to proceed with the calculations for h_o. The relationship of Rogers and Mayhew [3] based on the film temperature was used to determine the inner Nusselt number, Nu_i, of the coil (this is similar to the method of Ali [2]):

$$Nu_i = 0.021\, Re^{0.85}\, Pr^{0.4} (r_i/R)^{0.1} \tag{9}$$

Since the wall temperature was not known, the film temperature used to evaluate the Reynolds number, Re, and the Prandtl number, Pr, was also unknown. Therefore an iterative approach was used to determine the wall temperature and the inner Nusselt number simultaneously. A first approximation was made for the average wall temperature and was used to calculate the film temperature. All properties for the Reynolds number and the Prandtl number were evaluated at the film temperature and the Nusselt number was calculated using Eq. (9). The inside heat transfer coefficient, h_i, was then determined from

$$h_i = \frac{Nu_i\, k}{2 r_i} \tag{10}$$

The wall temperature, T_w, was then determined from

$$T_w = \frac{q}{h_i A_i} + T_{bulk} \tag{11}$$

T_{bulk} and A_i are the average bulk temperature (average of the inlet and outlet temperatures) of the processing fluid and the inside surface area of the coil, respectively. The average bulk temperature was based on the average of the inlet and outlet temperatures, which were both bulk temperatures. The newly calculated wall temperature was then used as a second approximation of the temperature and the iteration was repeated until convergence was obtained. The outside heat transfer coefficient was then calculated from the thermal resistance equation.

Outside Nusselt numbers, Nu_o, were then calculated using the outer heat transfer coefficient. Different characteristic lengths were used in the Nusselt number calculations to determine which length best fits the data. The corresponding Rayleigh number was based on the same characteristic length. The characteristic lengths used were:

(1) the overall length of the coil,
(2) the diameter of the tube,
(3) the diameter of the coil,
(4) the height of the coil (including space between coil turns), and
(5) as a normalized length.

The normalized length was calculated by assuming the coil as a cylinder. The normalized length was the outer surface area of this cylinder divided by the total tube length. A power law relationship was developed for the Nusselt number as a function of the Rayleigh number for each of the characteristic lengths used.

5. Development of a prediction model

The relationships developed for the Nusselt number as power function of the Rayleigh number was used in the development of a prediction model. The objective of the prediction model is to predict the outlet temperature from a coil given the inlet temperature, bath temperature, coil dimensions, and fluid flow rate through the coil. Data from coil 4 was used to validate the prediction model. A double iterative method was used in the prediction model to determine the outlet temperature from the coil and this was compared to outlet temperature measurements made on the fourth coil.

The following steps were used to write a code in Visual Basic 6.0™ that predicted the outlet temperature using the relationship for the inside Nusselt number of Rogers and Mayhew [3] and the outside Nusselt number relationship developed in this work (a fully developed flow chart is shown in Fig. 2):

(1) Input coil dimensions, mass flow rate, inlet and bath temperatures.
(2) Input initial approximation of the wall and outlet temperatures.
(3) Calculate film temperature and fluid properties at film temperature. Calculate the heat flux based on inlet and outlet temperatures, mass flow rate and the specific heat.
(4) Calculate Nusselt number (Nu_i) from Rogers and Mayhew [3].
(5) Calculate a new wall temperature using h_i from Nu_i determined in step 4 and the heat flux from step 3.

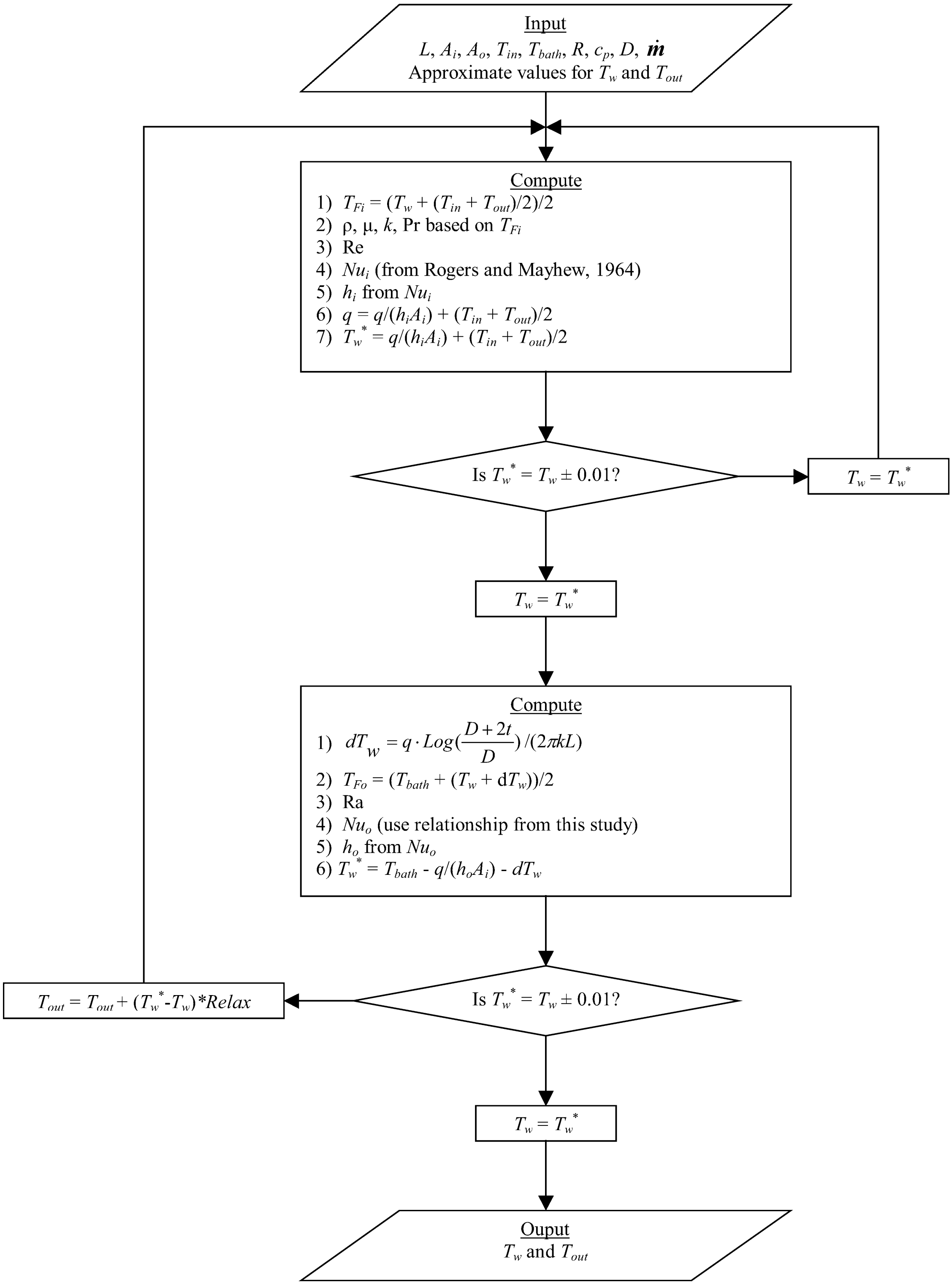

Fig. 2. Flowchart of the prediction model.

(6) Using the new wall temperature as the next approximation temperature, return to step 3 until convergence (until no change in successive wall temperatures).
(7) Calculate outside fluid properties at outside film temperature.
(8) Calculate Rayleigh and outside Nusselt numbers. Nusselt numbers are based on the models developed in this work.
(9) Using h_o and the heat flux, calculate expected wall temperature.

(10) Update outlet temperature by adding the difference between the calculated wall temperature of step 5 and the predicted wall temperature of step 9, multiplied by a relaxation factor.
(11) Return to step 3 using the new outlet temperature from step 10 and the new wall temperature from step 5. Repeat until convergence.

6. Results and discussion

The outside Nusselt number and the Rayleigh number were calculated for the five different characteristic lengths. The length of the tube, the height of the coil and the normalized length all produce relatively strong correlations between the Nusselt number and the Rayleigh number using a power law equation

$$\boldsymbol{Nu} = \boldsymbol{a}(\boldsymbol{Ra})^{\boldsymbol{b}} \tag{12}$$

Values for a and b, along with the correlation coefficient, are given in Table 2. Of the three models that had strong correlations, the coil height and the normalized length were the two that are preferred for this study and for future studies. Basing the characteristic length on the total tube length does not take into consideration the pitch, tube diameter, or coil diameter. Further work needs to be done for these types of correlations over a wider range of tube and coil diameters, as well as the number of turns.

Each of these three correlations was then used in the prediction model, to see which correlation would best predict the outlet temperature. The procedure for predicting the outlet temperature was written in Visual Basic 6.0™ code. All fluid properties were recalculated for each loop based on the updated film temperature and were expressed as functions of temperature. It was necessary to add a relaxation factor into the updating of the outlet temperature, as the solution would sometimes diverge if this were not included. Addition of a relaxation factor does not change the final temperature prediction; it only increases the number of iterations necessary to obtain a converged solution.

A method of least squares was used to develop a linear correlation between the predicted outlet temperatures and the measured outlet temperatures. The correlation coefficients obtained for these were 0.9871, 0.9988, and 0.9965, for the characteristic lengths of total tube length, coil height, and normalized length, respectively. The predicted outlet temperatures are plotted against the measured outlet temperatures (Fig. 3) for the case of coil height. A linear regression was performed on the data. Though the correlation coefficient is high (0.9988), the linear fit does not have the desired slope of unity. The predictions tended to be over the measured values at lower outlet temperatures and under the measured values at higher outlet temperatures. However, the largest temperature difference between the predicted and the measured was 3.5 °C. Average temperature difference was 1.3 ± 1.2 °C. These differences can be assumed to be within reason. The temperature measurements for the inlet, outlet and bath temperatures were measured at ± 2.2 °C for type K thermocouples, not including any other deviations in the data acquisition system. The relationship of Rogers and Mayhew [3] was for a steam heated wall boundary condition, whereas this work did not have the same boundary conditions. This is expected to result in a slight error. The model development was done on only three coils; more detailed work needs to be done to obtain better correlations between the Nusselt number and the Rayleigh number as a function of the coil dimensions. It is also difficult to use one characteristic length that will account for many different dimensions of the coil, including tube radius, coil radius, pitch, total tube length and number of turns. However, despite these sources of error, the predictions were reasonable and demonstrate that this framework for predicting outlet temperature holds promise. To make this procedure more accurate, two points nee further investigation. The first is the inside heat transfer characteristics. Nusselt number relationships that take into account the boundary conditions in a fluid-to-fluid heat exchanger need to be developed. Secondly, the outside heat transfer characteristics need to be studied in much more detail for the same reasons. Once these two areas are better understood, the framework presented in this study will benefit those that

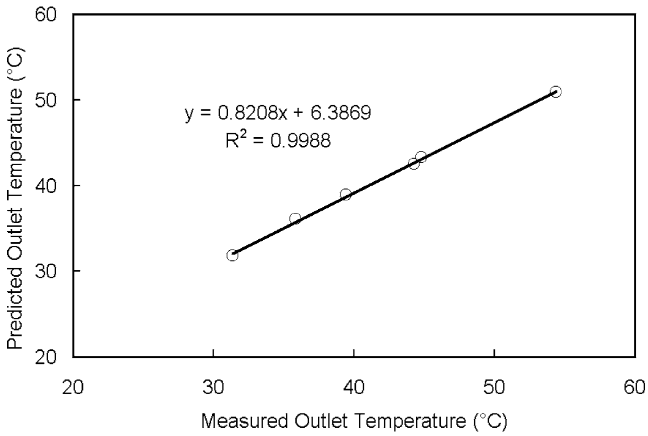

Fig. 3. Predicted versus measured outlet temperatures for coil 4 using the coil height as the characteristic length in the Nusselt–Rayleigh correlation.

Table 2
Nusselt number (Nu_o)–Rayleigh number (Ra) correlation results ($Nu_o = a(Ra)^b$)

Characteristic length	a	b	Rayleigh range	Correlation coefficient
Tube length	0.009759	0.3972	5×10^{14}–3×10^{15}	0.8684
Coil height	0.0749	0.3421	9×10^{9}–4×10^{11}	0.9306
Normalized length	2.0487	0.1768	2×10^{6}–3×10^{9}	0.9341

are trying to properly design and size helically coiled heat exchangers.

7. Conclusion

Different characteristic lengths were used for the correlation of the Nusselt number and the Rayleigh number. The coil height is considered as the best representation for a vertical coil in this study. Though this model did not have the highest correlation coefficient, it was the model that best predicted the outlet temperatures in the prediction model.

The prediction procedure used in this study shows promise as a method of predicting the outlet temperature from a coil given the inlet temperature, bath temperature, and coil dimensions. However, further work is required to develop better Nusselt number relations to cover a wider range of sizes and configurations. A similar approach could be used for sizing helically coiled heat exchangers by specifying an outlet temperature and using the iterative approach to determine the required surface area.

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